R-star in Transition Economies: Evidence from Slovakia[1]

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Abstract

The aim of this paper is to estimate the equilibrium real interest rate in Slovakia by means of a semi-structural unobserved components model. The equilibrium real interest rate is understood here as a short-term, risk-free real interest rate consistent with output at its potential level, and inflation at its target level after the effect of all cyclical shock have disappeared. Contribution to the literature is in two ways: (i) development of a modelling framework for small, open, and converging economies which can be used for other transition economies, and (ii) assessment of the adoption of the euro and its effect on the equilibrium real interest rate. Based on the estimates, the equilibrium real interest rate fell from the positive pre-euro (also pre-crisis) level into to the negative territory.

Keywords: *equilibrium real interest rate, unobserved components model, transition economy, monetary policy*

JEL Classification: E43, E52, E58

DOI: https://doi.org/10.31577/ekoncas.2020.08.01

Introduction

The equilibrium real interest rate sets a benchmark for assessing the stance of monetary policy and is understood here as a short-term, risk-free real interest rate consistent both with output at its potential level, and inflation at its target level after the effect of all cyclical shocks have disappeared.

Equilibrium real interest rate as an unobserved variable must be extracted from the data. A number of concepts are currently used to model the equilibrium real interest rate, with differences based on the analysed time horizon. First,

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[1] The research was carried out within the ECB Working Group on Econometric Modelling, specifically the Natural Rate of Interest team. Results were presented at the 2018 Slovak Economic Association Meeting in Bratislava. The views and results presented in this paper are those of the author and do not necessarily represent the official opinion of the National Bank of Slovakia.

OLG (overlapping generations) models (Carvalho, Ferrero and Nechio, 2016) are used in the long-run. Second, statistical filters such as Hodrick-Prescott filter, error correction models, and unobserved components models (Berger and Kempa, 2014) model the medium-term component. Finally, DSGE models or full structural models (Neri and Gerali, 2019) capture business cycle characteristics of equilibrium real interest rate on a monetary policy horizon of 1 – 2 years.

The unobserved components modelling framework (Berger and Kempa, 2014) was used for transition economies (Grafe, Grut and Rigon, 2018) and earlier for Slovakia (Benčík, 2009a;b), however without explicit consideration of the transition process demonstrated through the trend appreciation of the exchange rate. We try to fill this gap by modifying existing framework of unobserved components models to account for the characteristics of a transition economy.

Our contribution to the literature is twofold: (i) development of a modelling framework for small, open, and converging economies which can be used for other transition economies; and (ii) assessment of the adoption of the euro and its effect on the equilibrium real interest rate.

The rest of the paper is organized as follows: the first section briefly summarizes the relevant literature, section 2 lays out the model and describes the data used in estimation, section 3 discusses the estimated parameters of the model and assesses both the transition process and the evolution of the equilibrium real interest rate and its drivers. Finally, the last section concludes.

1. Literature Review

The equilibrium[2] real interest rate sets a benchmark for assessing the stance of monetary policy, with policy being expansionary (contractionary) if the short-term real interest rate is below (above) the equilibrium real interest rate. This topic is extremely relevant today as many advanced economies have approached the ZLB with their nominal policy rates in the wake of the Global financial crisis. Despite this importance, equilibrium real interest rate is not directly observed and must be derived from the data.

The current research has not identified a unified approach to model the equilibrium real interest rate, but the methods can be generally classified into three broad categories depending on the horizon over which one wants to study the relationship of the equilibrium real interest rate and the real economy: (i) OLG models represent the first category and are used to study demographic changes (Krueger and Ludwig, 2007; Lee, 2016; Carvalho, Ferrero and Nechio, 2016) or

[2] We use the term equilibrium, natural, neutral, “R-star”, or r^* interchangeably.

income inequality effects on the equilibrium real interest rate in the long run. According to the neoclassical growth model, the equilibrium real interest rate is in the long run driven by labour force growth, technological progress and the households time preference. Results for the euro area by Bielecki, Brzoza-Brzezina and Kolasa (2018) show that ageing, higher life expectancy, and changing composition of age cohorts have had an average dampening effect on the equilibrium rate of up to 1% over the last 30 years. Based on current trends, this decline will sustain by another 0.5% until 2030; (ii) statistical filters, error correction models, and unobserved components models (Laubach and Williams, 2003; Holston, Laubach and Williams, 2017; Berger and Kempa, 2014; Pedersen, 2015) are among the second category of methods and are usually employed to extract the medium-term component. They study the evolution of the macroeconomic equilibria and decompose the observed macroeconomic variables into their trend and cycle components. These estimates suggest that the average value of the equilibrium real interest rate before the Global financial crisis was around 2% but it turned negative afterwards. The main drivers have been slowdown in productivity growth (possibly as a consequence of unfavourable demographics) and higher risk aversion; (iii) as it is now a common practice in many central banks, DSGE models of Smets and Wouters (2007)'s type with financial frictions are widely used for monetary policy analysis. In this class of models, it is possible to extract the equilibrium real interest rate on the monetary policy horizon. These models, such as Neri and Gerali (2019), have also identified decline in equilibrium rates from positive values into the negative territory after the financial crisis. Not only aforementioned factors, but also the risk premium shocks and other financial factors or frictions may have played a substantial role.

Modelling approach from the second category has been used most recently by Stefański (2018) and Grafe, Grut and Rigon (2018) for the other V4 countries except for Slovakia (Czech Republic, Hungary, Poland), CEE countries (e.g. Romania), and other emerging markets (Israel, Turkey, South Africa, Russia). Stefański (2018) found that the equilibrium real interest rate fell from around 3 – 4% to negative levels immediately after the Global financial crisis and slightly rebounded to 1% in recent years. Similar conclusion can be drawn from Grafe, Grut and Rigon (2018) results. Regarding the most important factors, Stefański (2018) identified slowdown in productivity growth, whereas Grafe, Grut and Rigon (2018) found little role for productivity growth. Bigger part of the neutral rate dynamics can be explained by common global component which is modelled in their model as US neutral rate extracted from Laubach and Williams (2003) model.

Finally, Benčík (2009b) used, to some extent, approaches from the first and second category for Slovakia. He also documented the fall in the neutral real interest rate, however, on a limited sample from 1997 to 2007. Naturally, without the full assessment of the effects of accession to eurozone, without identifying underlying factors, and without explicitly modelled transition process.

In this paper we use the approach from the second category, as it is in our view the most convenient way to estimate equilibrium real interest rate in the small open economy. Closed economy workhorse model of Laubach and Williams (2003), extended by Berger and Kempa (2014) to account for the open economy issues, is modified to account for the transition process of planned economies to market-based.

2. The Model and the Data

2.1. The Model

The model used in this paper is an open economy version of Laubach and Williams (2003) model as proposed in Berger and Kempa (2014) and applied in Pedersen (2015). The model is estimated using four variables: real output, y_t, real interest rate, r_t, real effective exchange rate, q_t,[3] and inflation, π_t. In this semi-structural model, equilibrium variables are modelled as random walks, while the temporary components are related through the standard aggregate demand and aggregate supply curves. In addition, open economy aspect is captured through the evolution of the real effective exchange rate.

Observed output, real interest rate, and real effective exchange rate can be decomposed into their equilibrium levels (denoted with an asterisk) and gaps (denoted with a tilde):

$$y_t = y_t^* + \widetilde{y_t} \tag{1}$$

$$r_t = r_t^* + \widetilde{r_t} \tag{2}$$

$$q_t = q_t^* + \widetilde{q_t} \tag{3}$$

Usually, inflation is modelled in a traditional backward-looking or “accelerationist” manner (see, for example, former OECD approach to estimating Phillips curves and unemployment gaps in Guichard and Rusticelli, 2011, or as it is standard in this stream of literature in Laubach and Williams, 2003 or Holston, Laubach and Williams, 2017). In this specification, inflation is a function of inflation drivers related to demand factors (unemployment gap, output gap, ...),

[3] Exchange rate is defined as foreign currency per unit of home currency (English way), so increase in exchange rate means appreciation.

supply factors (import price inflation, oil price inflation, changes in indirect taxes, …), and inertia represented by an autoregressive distribute lags of past inflation:[4]

$$\Delta\pi_t = \beta_\pi (L) \Delta\pi_{t-1} + \beta_d * demand\ factors + \beta_s * supply\ factors + \epsilon_t^\pi \qquad (4a)$$

However, when the backward-looking specification (4a)[5] is estimated over a recent sample period, the coefficient on the unemployment gap β_d is usually not statistically significant for most OECD countries. Rusticelli, Turner and Cavalleri (2015) and others explain this phenomenon of "flattening of the Phillips curve" with better anchored inflation expectations in the inflation targeting monetary policy framework. A central bank with credible inflation target attracts inflation expectations, therefore decreasing inflation persistence and reducing the effectiveness of the current rate of inflation to predict the next period rate of inflation. This has been recognized in the literature (Coibion and Gorodnichenko, 2015) as the main explanation for more stable inflation and for the absence of significant disinflation after the Global financial crisis when the unemployment fell substantially. The anchored expectations Phillips curve can be written as:

$$\begin{aligned}\Delta\pi_t = \beta_{att} \left(\pi_{t-1} - \pi^e\right) + \beta_\pi (L) \Delta\pi_{t-1} + \beta_d * demand\ factors + \\ + \beta_s * supply\ factors + \epsilon_t^\pi\end{aligned} \qquad (4b)$$

where π^e are inflation expectations. Rusticelli, Turner and Cavalleri (2015) found, that in the sample of OECD countries from 1998[6] to 2014, estimates of π^e are consistent with the expectations anchored at the official inflation target $\pi^e = \pi^{IT}$. Moreover, coefficients on the unemployment gap β_d are statistically significant for all OECD countries and the curve provide a better fit in terms of R^2 as well.

In this paper we use the anchored expectations Phillips curve in two stages. In the first stage we estimate equation (4b) only with first two terms on the right-hand side. Then, in the second stage, residuals from the first stage, called inflation drivers (demand and supply factors from (4b)), enter the model as fourth observation equation in the form:

$$inflation\,drivers_t = \beta_y \tilde{y}_{t-1} + \beta_q \Delta q_{t-1} + \epsilon_t^\pi \qquad (4c)$$

[4] Δ ensures that the sum of lagged coefficients on inflation is equal to 1. See, for example, Hooper, Mishkin and Sufi (2019), Turner et al. (2019), or Rusticelli, Turner and Cavalleri (2015).

[5] In the following text, all shocks are white noise processes with standard deviations, in this case σ_π, to be estimated.

[6] This starting date was formally tested in Rusticelli, Turner and Cavalleri (2015) as a starting date at which inflation expectations became well-anchored at inflation target for a broad set of countries.

where domestic activity creates price pressures through the output gap ($\beta_y > 0$), and the first difference in the real effective exchange rate captures the impact of foreign activity and inflation on the domestic price developments. An appreciation of the exchange rate, ($\Delta q_t > 0$), implies that foreign goods are cheaper, and hence, the rate of inflation should fall ($\beta_q < 0$).

Aggregate demand (IS curve) relates the output gap to the real interest rate gap and the real effective exchange rate gap:

$$\tilde{y}_t = \alpha_y \tilde{y}_{t-1} + \alpha_r \tilde{r}_{t-1} + \alpha_q \tilde{q}_{t-1} + \epsilon_t^{\tilde{y}} \tag{5}$$

As in standard macroeconomic models, positive realization of the real interest rate gap is associated with dampening of economic activity ($\alpha_r < 0$). Real exchange rate above its equilibrium value means overvaluation of the home currency, worsening the current account and thus lowering the level of economic activity below potential ($\alpha_q < 0$). Potential product is assumed to follow a local level model

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_t^{y^*} \tag{6}$$

with stochastic drift

$$g_t = g_{t-1} + \epsilon_t^g \tag{7}$$

which is assumed to represent productivity growth in the economy.

The relationship for the equilibrium real interest rate has its roots in the standard optimal growth or neoclassical model of the Ramsey (1928) type. More recently, Rachel and Smith (2017) use the following formulation:

$$r^* = (1/\sigma) * g + \alpha * n + \beta \tag{8a}$$

where σ denotes the intertemporal elasticity of substitution in consumption, g is the rate of labor-augmenting technological change, α is the coefficient on the rate of population growth, n is the rate of population growth, and β is the rate of time preference. Following Laubach and Williams (2003), Berger and Kempa (2014), and Pedersen (2015), we model the real equilibrium interest rate as:[7]

$$r_t^* = cg_{t-1} + z_{t-1} \tag{8b}$$

A lower potential growth, either because of lower productivity or population growth, will tend to lower $r^* (c > 0)$. Regarding the second component, more

[7] As was done in other studies, r^* can be modelled only as a random walk process as is the case for other equilibrium variables. But as we want to identify structural factors behind it, we use the mentioned specification.

patient agents in the economy will tend to lower r^* as well. The second component in (8b) tries to capture this “patience” and is modelled as a random walk.[8]

$$z_t = z_{t-1} + \epsilon_t^z \tag{9}$$

Berger and Kempa (2014) and Pedersen (2015) use a random walk process for the equilibrium real effective exchange rate and an AR(p) process for the real effective exchange rate gap. The theory behind these relations is basically Power Purchasing Parity (PPP). In the long run, for countries at comparable levels of development, the level of the real exchange rate should be equal to 1. The price levels in the home and the foreign country should be equal when expressed in the same currency unit. Because of differences in the tax system, wage policies, trade barriers, and other imperfections the real effective exchange rate should fluctuate in a band around 1. Authors apply this theory for developed economies of Canada and Denmark in samples starting in 70s.

Egert, Halpern and MacDonald (2006) in a comprehensive study propose and evaluate alternative methods for modelling the equilibrium real exchange rate in transition economies. We adopt their theory of trend adjusted PPP. This theory tries to explain trend appreciation in transition economies through the existence of the non-tradable sector and the Balassa-Samuelson effect.[9] More specifically, they stress two important factors behind trend adjusted PPP: (i) initial undervaluation of transition economies, and (ii) trend appreciation of the tradable sector’s real exchange rate related to the transformation process.

Taking into account trend appreciation in the transition economy, potentially as a result of the aforementioned factors, we allow the equilibrium real exchange rate to grow over time. Equilibrium real effective exchange rate follows a random walk with stochastic drift μ_t:

$$q_t^* = q_{t-1}^* + \mu_{t-1} + \epsilon_t^{q^*} \tag{10}$$

$$\mu_t = \mu_{t-1} + \epsilon_t^{\mu} \tag{11}$$

Temporary deviations from this equilibrium level are modelled as an AR(1) process:

$$\widetilde{q_t} = d_q \tilde{q}_{t-1} + \epsilon_t^{\tilde{q}} \tag{12}$$

[8] In their original paper, Laubach and Williams (2003) use an AR(2) process in addition to the random walk process for the variable *z*. Nevertheless, they found similar results in terms of coefficient *c*, which was always near unity. In the following literature, authors use mainly random walks processes.

[9] See Balassa (1964) and Samuelson (1964).

Finally, the real interest rate gap is related to the real effective exchange rate gap such as:

$$\tilde{r}_t = \gamma \tilde{q}_{t-1} + \kappa_{t-1} \tag{13}$$

$$\kappa_t = \rho \kappa_{t-1} + \epsilon_t^{\kappa} \tag{14}$$

Berger and Kempa (2014) call the relationship (13) as an interest rate-exchange rate nexus or as a real interest rate parity condition in gaps as in Pedersen (2015). Intuitively, if the exchange rate is overvalued or above its long-run level $(\tilde{q} > 0)$, investors will expect a possible future depreciation because of mean-reverting nature of the real exchange rate gap in (12). Capital outflows will occur, and the real interest rate decreases $(\tilde{r} < 0)$, which means that $(\gamma < 0)$. However, if the central bank uses a real effective exchange rate as an operating target, it may choose to react to the expected depreciation of the real effective exchange rate gap by rising interest rates and dampen or reverse capital outflows to stabilise the exchange rate. In that case $(\gamma > 0)$.[10] The error term κ_t in (13) captures all factors which may impinge on the interest rate-exchange rate nexus, such as time-varying risk premia or any other distortions in international capital markets.

2.2. Estimation Methodology

The model described in observation equations (1), (2), (3), and (4c) and state equations (5), (6), (7), (8b), (9), (10), (11), (12), (13), and (14) is converted into the Gaussian state space form:

$$Y_t = Z\xi_t + \varpi_t \tag{15}$$

$$\xi_{t+1} = T\xi_t + K\vartheta_t \tag{16}$$

Equation (15) is the observation equation in a matrix form. Y_t is a $p \times 1$ vector of p observed variables. Equation (16) is a state or transition equation in a matrix form. ξ_t is a $m \times 1$ vector of m unobserved states. The vector ϖ_t represents measurement errors and ϑ_t represents structural shocks. Both these innovations are vector white noises with $E\left[\varpi_t \varpi_\tau^{'}\right] = R$ for $t = \tau$ and 0 otherwise, and $E\left[\vartheta_t \vartheta_\tau^{'}\right] = Q$ for $t = \tau$ and 0 otherwise.

[10] Interest rates together with exchange rates form the so-called MCIs (Monetary Condition Indexes), which may guide monetary policy in small open economies. Gerlach and Smets (2000) estimated the responses of the central banks to exchange rate movements for open economies and found that the Reserve Bank of Australia does not appear to respond, the Bank of Canada and the Reserve Bank of New Zealand, who use the MCIs as an operating target, do respond strongly to movements in the exchange rate. Berger and Kempa (2014) identified similar significant response for Canada, but Pedersen (2015) did not find any significant response for Denmark.

The goal is to estimate the vector of parameters θ hidden in matrices *Z*, *R*, *T*, *Q* and to recover unobserved variables in ξ_t. This is effectively done using the Kalman filter which evaluates the likelihood function of a state space model and forms the estimates of unobservable states. As Stock and Watson (1998), Laubach and Williams (2003), or Mésonnier and Renne (2007) pointed out, the problem with this approach is that, if the model is simultaneously estimated via the ML, the variance of one of the shocks (probably the shock to the variable with highly persistent changes such as the growth rate of potential output) will be biased towards zero.

This problem is usually solved with the Stock and Watson (1998)'s median unbiased estimator resulting from the multi-step ML estimation or, as we do in this paper, employing a Bayesian approach. Bayesian approach has a number of advantages (see Griffoli, 2007) such as: (i) fits the complete, solved model, as opposed to particular equilibrium relationships; (ii) down-weighting the likelihood function in regions of the parameter space that are inconsistent with our prior beliefs; (iii) adds curvature where the likelihood function is flat. Moreover, as Fernández-Villaverde (2010) pointed out, this approach is useful for transition economies where the data issues are considerable, and the prior information is important. Bayesian estimation consists of setting the prior density function $p(\theta)$ for each estimated parameter and the evaluation of the likelihood function $L = (\theta \mid Y_t)$ through the Kalman filter under the assumption of conditionally independent Gaussian projection errors. This gives, in the log terms, posterior kernel:

$$lnK(\theta|Y_t) = lnL(\theta|Y_t) + lnp(\theta) \tag{17}$$

The posterior kernel (17) is a nonlinear and complicated function of deep parameters of the model and we cannot obtain explicit form of it. The mode is obtained by maximizing the posterior with respect to θ. Since we are more interested in the mean and variance of this distribution, we must rely on sampling methods which usually start from the posterior mode. In this paper we use the popular Monte Carlo Markov Chain Metropolis-Hastings algorithm. An and Schorfheide (2007) characterise this algorithm as an algorithm, which constructs a Gaussian approximation around the posterior mode and uses a scaled version of the asymptotic covariance matrix as the covariance matrix for the proposal (jumping) distribution.

2.3. Data

To illustrate the application of the model, we use quarterly data for Slovakia from 1994Q2 to 2019Q3 (102 observations) taken from the Statistical Office of the Slovak Republic (SOSR), National Bank of Slovakia (NBS), European Central

Bank (ECB), and Bank for International Settlements (BIS). All data transformations can be found in Table 1[11] and are depicted in Figure 1.

T a b l e 1

Data Used in Estimation

	Name	Transformation	Original series	Source
y_T	Output	$100*log(RealGDP)$	Real GDP, EUR, SA	SO SR
π_t	Inflation	$400*log\left(HICP_t^{core} / HICP_{t-1}^{core}\right)$	HICP core[a], index, NSA	NBS
r_t	Real interest rate	$i_t - \pi_t^e$	3M Interbank rate[b], %	NBS/ECB
q_t	Real effective exchange rate		REER(Broad)[c], index	BIS

Notes: If necessary, data were seasonally adjusted and converted to quarterly frequency using the average value over the period method; i_t is a 3M interbank rate (official policy rate until 2000Q1) expressed on a 365-day basis; inflation expectations π_t^e are approximated as a four-quarter moving average of past inflation; real effective exchange rate is deflated by the CPI index; HICP core excluding food, energy, alcohol and tobacco (CPI until 1996M1).

Source: SOSR, NBS, ECB, BIS.

When estimating the first stage of anchored expectations Phillips curve (4b),[12] the level of expected inflation in the whole sample is at 3.4%. As there is no official inflation target in Slovakia, we will refer to this parameter as inflation attractor.[13] Clearly, there is strong evidence of the structural change in this attractor. In the visual inspection of inflation (Figure 1, Panel (d), solid line) it seems that the break had occurred broadly at the time of joining the European Union and the European System of Central Banks in May 2004. Around the same period, National Bank of Slovakia adopted inflation targeting monetary policy regime and committed itself to the adoption of the euro in 2009. Bai and Perron (1998) structural break test identified the break date to be in 2004Q2.[14] In

[11] Interbank rate represents monetary policy element together with credit risk. In normal times this credit spread is stable and monetary policy stance in macroeconomic models is usually captured by interbank rate (Walsh, 2017). However, in periods of stress, as was observed in Slovakia at the end of 90s, the spread widens. That is why we use official policy rate for this period.

[12] We use dummy variables for dates 1994Q4, 1999Q3, and 2002Q1. Based on statistical significance, we do not use higher lags of inflation than 1.

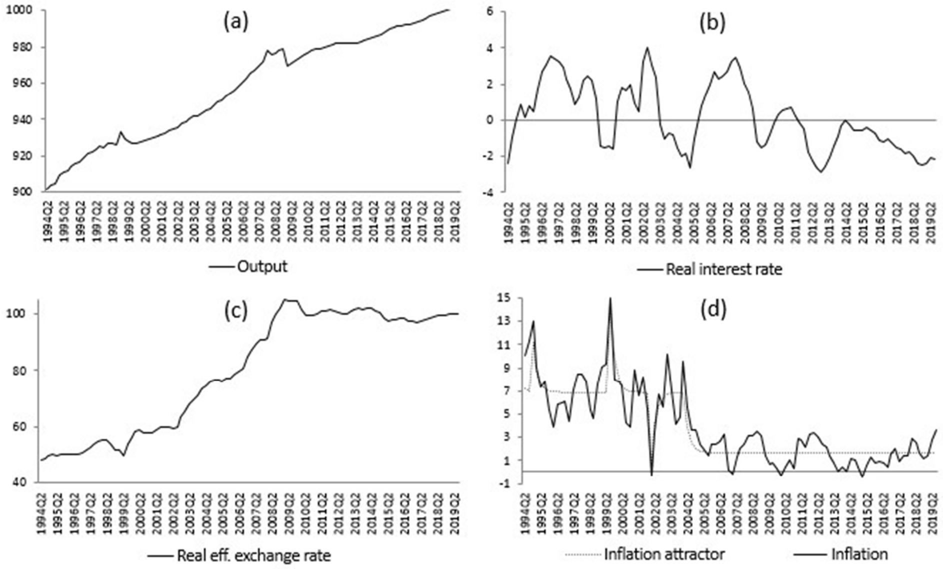
[13] In the more recent study, Turner et al. (2019) analyse if the official inflation target is still the appropriate attractor, in the light of the recent experience of inflation remaining below official targets despite the continued recovery. They conclude that there are better inflation attractors (such as surveys of inflation expectations or the official target adjusted for the slippage of expectations) than the official targets. For Slovakia we have reliable data for inflation expectations only from 2002, that is why we use aforementioned, empirically determined, inflation attractor.

[14] First, Bai and Perron (1998) suggest checking if there are any structural breaks at all. The so-called WD_max and UD_max statistics rejects the null hypothesis of no structural breaks in the attractor against an alternative of maximum of 3 breaks at standard confidence levels. Next, sequential analysis rejects the null hypothesis of no breaks against the alternative of one break. However, procedure does not reject the null of one break against the alternative of two breaks. The break date for the attractor has been identified to be in 2004Q2.

the first inflation regime, inflation was oscillating around 6.9% and in the second regime around 1.7% (Figure 1, Panel (d), dashed line). Regarding the parameter β_{att} in (4b), which is statistically significant, every quarter almost 60% of the deviation of inflation from its attractor is on average corrected in the next period.

F i g u r e 1

Data Used in Estimation



Note: Data used in estimation, see Table 1.

Source: SO SR, NBS, ECB, BIS, and author's own computations.

3. Results

3.1. Prior Distribution of the Parameters

In setting the priors we follow Berger and Kempa (2014) and Pedersen (2015) who assume Gaussian prior distributions for all parameters except for the standard deviation parameters which have Inverse gamma distribution (Table 2).

For standard deviation parameters in structural shocks (16) and observation errors (15) we use somewhat higher prior means than in Berger and Kempa (2014) or Pedersen (2015) to account for potential higher variability in Slovak data. For example, the highest values have structural shocks in the output gap and exchange rate equation as well as observation error in inflation drivers equation. We choose lower values for equilibrium processes. Moreover, we set standard deviations of these priors to equal infinity which is common in the literature

(Adolfson et al., 2013), except for the shock to the potential output. The reason is that for this kind of a state space model it is hard to disentangle shocks to the potential output from the shocks to the output gap as it is documented in Mésonnier and Renne (2007). Authors usually calibrate this ratio, but we rather set a tighter prior for the innovation in the potential output equation to match the variability of the official estimate of the output gap and potential product by the National Bank of Slovakia.[15] All autoregressive parameters are set to 0.75.

We do not impose any strong beliefs on priors for structural parameters. Still, previous maximum likelihood estimates for Slovakia in Benčík (2009b) in the shorter sample (1997 – 2007) as well as economic theory do provide some approximate values for model parameters. Interest rate and exchange rate together form monetary condition index, which in the case of a small open economy, gives more weight to the exchange rate.[16] The effect of interest rate on the output gap α_r should be, by economic theory, negative. We set $\alpha_r = -0.05$ which is roughly the estimated value from Berger and Kempa (2014) and from Benčík (2009a). Because of the nature of the Slovak economy, the higher prior mean is used for the effect of the exchange rate on output, ($\alpha_q = -0.25$). The slope of the Phillips curve β_y is expected to be positive, but rather small as was discussed in Rusticelli, Turner and Cavalleri (2015), so we set the prior mean to be ($\beta_y = 0.05$). The direct effect of exchange rate appreciation on inflation (the indirect effect is through its effect on the output gap) is a product of two factors. First is the exchange rate pass-through on import prices, the other one is the import share in the consumption basket. Goldberg and Campa (2010) found, that the average pass through in developed countries is around 0.15, the same value as in Benčík (2009b), but lower than the value set in this study ($\beta_q = -0.25$) and in Berger and Kempa (2014) and Pedersen (2015) for Canada and Denmark. Parameter linking potential output growth to the equilibrium real interest rate, c, is often not sufficiently identified in data. Laubach and Williams (2003) estimate this parameter to be around 1, but other authors (Mésonnier and Renne, 2007 or Holston, Laubach and Williams, 2017) use only its calibrated value. If we look at r^* from the optimal growth model perspective (8a), the link between the two depends on the intertemporal elasticity of substitution parameter. Havranek et al. (2015) in a meta-study found that the global average for this parameter is 0.5, which means one for-two mapping from productivity growth to r^*. On the other hand, Hamilton et al. (2016) argue that this relationship is much more tenuous than widely believed value 1. So the prior mean at 1 (4 for annualised data) with wide variance seems

[15] National Bank of Slovakia Macroeconomic Database available at: <https://www.nbs.sk/en/monetarypolicy/macroeconomic-database/macroeconomic-database-chart>.

[16] See Gerlach and Smets (2000).

appropriate. Finally, the prior mean for the parameter γ, which links the real effective exchange rate gap to the real interest rate gap, is set to zero to test the interest rate-exchange rate nexus in the case of Slovakia.

3.2. Posterior Distribution of the Parameters

To find the mode of the estimated parameters we use the continuous simulated annealing global optimisation algorithm.[17] Metropolis-Hastings has been replicated 600 000 times in 5 parallel blocks and the first 40% of draws have been discarded before computation of the posterior statistics. The scale parameter of the jumping distributions covariance matrix has been tuned to 0.52 to obtain the average acceptance ratio of proposed parameters of 23.38%.

The last three columns in Table 2 show the posterior mean and the 10% and 90% percentiles of the posterior distribution of all estimated parameters and standard deviations.[18] The persistence of the output gap (α_y = 0.78) is smaller than the persistence parameters in the exchange rate equations (d_q = 0.89) and (ρ = 0.82). This is not surprising for the Slovak economy, which experienced officially only two cycles with sudden drop and quick rebound in economic activity. On the other hand, real effective exchange rate gap and other determinants in the interest rate-exchange rate nexus tend to deviate more persistently from their equilibrium levels.

Structural parameters in the IS curve (5) have expected signs, however, the effect of the interest rate gap (α_r = –0.13) is insignificant. Some empirical studies, such as Stracca (2010), have also found insignificant and even positive estimates. The effect of the real effective exchange rate gap is significant and negative (α_q = –0.24) and confirms the importance of the international competitiveness in the case of a small open economy. Slope of the Phillips curve (β_y = 0.08) is positive but not significant. This is not surprising. Aforementioned studies (Rusticelli, Turner and Cavalleri, 2015 or Turner et al., 2019), which find statistically significant slopes, use as a measure of slack unemployment gap. Hooper, Mishkin and Sufi (2019) argue, that unemployment gap generally yields a better statistical fit and the Phillips curve slopes are generally twice as large as those on output gap, consistent with Okun's law. The effect of exchange rate appreciation is significant and negative (β_q = –0.16), which means that foreign goods become

[17] Corana et al. (1987) and Goffe, Ferrier and Rogers (1994).

[18] The estimated prior-posterior distributions as well as the Brooks and Gelman (1998) convergence diagnostics of the Monte Carlo Markov Chains are available upon request in technical appendix. Based on statistical significance, we use mainly lagged variables in (t −1), except for the effect of real interest rate and exchange rate gap (t −2) in equation (5), and the effect of real exchange rate on inflation (t −3) in equation (4c).

cheaper as the home currency appreciates, and as they are part of the home consumption basket, home inflation falls. The link between the potential output growth and equilibrium real interest rate is 0.84 (annually c = 3.37), which is less than usually assumed, but consistent with estimates for other open economies. Finally, the parameter in the interest rate exchange rate nexus is negative (γ = –0.43), which means that appreciation of home currency is met with expectations of a subsequent depreciation and capital outflows, putting downward pressure on the real interest rate gap.

Table 2

Prior and Posterior Parameter Distributions

	Prior distribution			Posterior distribution		
Parameter	*Type*	*Mean*	*s.d.*	*Mean*	*10pct.*	*90pct.*
α_y	Normal	0.750	0.200	0.779	0.594	0.977
α_r	Normal	–0.050	0.150	–0.128	–0.327	0.070
α_q	Normal	–0.250	0.150	–0.239	–0.437	–0.040
σ_{y^*}	Inv. gamma	0.750	0.025	0.752	0.711	0.794
σ_g	Inv. gamma	0.150	inf.	0.102	0.042	0.162
$\sigma_{\tilde{y}}$	Inv. gamma	2.000	inf.	1.356	1.124	1.586
β_y	Normal	0.050	0.150	0.075	–0.024	0.178
β_q	Normal	–0.250	0.150	–0.163	–0.295	–0.030
σ_π	Inv. gamma	3.000	inf.	1.277	1.124	1.425
c	Normal	4.000	1.000	3.365	1.813	4.899
γ	Normal	0.000	0.500	–0.432	–0.663	–0.201
ρ	Normal	0.750	0.200	0.816	0.683	0.965
σ_z	Inv. gamma	0.700	inf.	0.318	0.178	0.457
σ_κ	Inv. gamma	1.500	inf.	0.505	0.367	0.640
d_q	Normal	0.750	0.200	0.885	0.792	0.996
σ_{q^*}	Inv. gamma	1.200	inf.	0.669	0.381	0.947
σ_μ	Inv. gamma	0.400	inf.	0.305	0.141	0.467
$\sigma_{\tilde{q}}$	Inv. gamma	2.000	inf.	0.873	0.634	1.110

Source: Author's own computations, Berger and Kempa (2014).

3.3. Posterior Distribution of the States

Figure 2 shows smoothed equilibrium and cyclical components of output (Panel (a) and (b)); real interest rate (Panel (c) and (d)); real exchange rate (Panel (e) and (f)); growth component in equilibrium real effective exchange rate and output (productivity growth) (Panel (g)); and inflation drivers (Panel (h)).

According to the output gap (Figure 2, Panel (b)), there were three episodes of overheating: (i) the second half of 90s; 2005 – 2012 period with a dip in 2009 caused by the Global financial crisis; and (iii) 2018-present. The first episode was caused primarily by home expansionary but unsustainable fiscal policy, the second one resulted from strong global growth and from positive effects of early implemented structural changes and massive foreign direct investment. Finally, the third episode stemmed from synchronised global growth (which reversed into synchronised slowdown, mainly as a result of trade uncertainty and maturing global growth cycle). At the same time, we can identify two periods of Slovak economy operating below potential: (i) 1999 – 2004 period, which was a period of deep structural reforms and stabilisation macroeconomic policies; and (ii) 2013 – 2017 period, which followed the double dip recession in eurozone and also the manufacturing slowdown in 2015 – 2016 in advanced economies. Price pressures (2, Panel (h)) broadly coincide with periods of positive output gap.

3.3.1. Transition Process

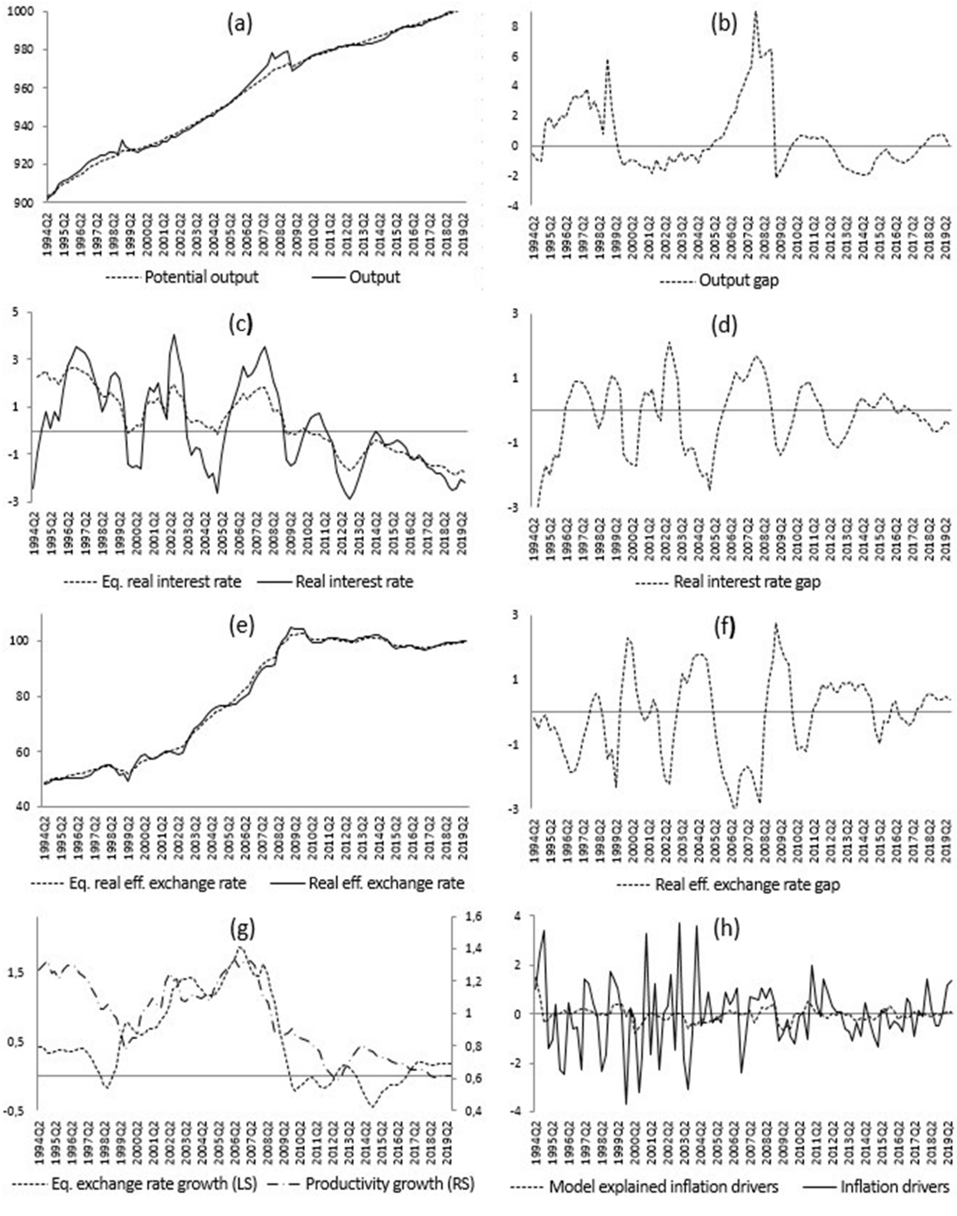
The evolution of the real effective exchange rate gap (Figure 2, Panel (f)) is related to domestic and foreign macroeconomic variables, while (trending) equilibrium real effective exchange rate (Figure 2, Panel (e)) captures the transition process of Slovak economy.

In terms of the real effective exchange rate gap, conclusions from Gylánik (2012) can be applied here. Exchange rate was undervalued from 1994 to 1997 what is characteristic for a transition economy entering the transformation process. Keeping the Slovak currency in the fixed exchange rate regime led to its slight overvaluation in 1997 – 1998. Growing home imbalances along with external shocks (Asian and Russian financial crisis and subsequent uncertainty in the exchange rate market) had demonstrated through double deficit. To fight these imbalances, a set of restrictive measures and transition to the floating exchange rate was undertaken which resulted in the negative gap in 1998 – 1999. Positive expectations about the future growth based on structural reforms led to overvaluation of the effective exchange rate from 1999 to 2001 and from 2003 to 2005.

On the other hand, uncertainty related to the parliamentary elections in 2002 and 2006 could potentially explain the related undervaluation. Real exchange rate became overvalued in 2009 because the fixation of the Slovak crown’s nominal exchange rate to euro had already taken into account the future equilibrium appreciation based on the continuing real convergence. Nevertheless, loss of independent monetary policy appeared to be not a problem for the stabilisation of the real effective exchange rate as it fell below the equilibrium the very next year. Rebound in the growth of Slovak economy (or not pronounced slowdown as elsewhere) seemed to explain overvaluation in 2011 – 2015 and in 2018. It should be noted, that after 2009 the variability of the exchange rate gap has decreased.

F i g u r e 2

Smoothed Equilibrium and Cyclical Components of Output, Real Interest Rate, and Real Effective Exchange Rate



Notes: Last data point 2019Q3.

Source: SO SR, NBS, ECB, EC, and author's own computations.

Regarding the transition process, Egert, Halpern and MacDonald (2006) point to two main explanations for the failure of PPP in transition economies, which are closely related to the nature of economic transformation from planned to market-based economies. The first one is related to the initial structural undervaluation of the transition economies' currencies. Authors argue that a large initial depreciation is needed to curb demand for foreign goods and currency, whereas price liberalisation yielding high inflation gives another motive to switch to foreign currency positions. Another reason is due to large uncertainty around the equilibrium exchange rate, and that policymakers rather prefer to undershoot the estimated equilibrium exchange rate. This could be the case also in Slovakia. Real exchange rate was undervalued in terms of cyclical factors towards the end of 90s (Figure 2, Panel (f)) and in the level of exchange rate (Figure 2, Panel (e)).

Following this initial undervaluation, real exchange rate of the tradable sector, and of the whole economy, tend to adjust (appreciate) towards the equilibrium, which is seen to be the second explanation. At the beginning of transition, both domestic and foreign consumers tend to prefer foreign goods. As the economic transformation gains momentum and productivity increase in the tradable sector, domestic economy becomes capable of producing growing number of goods of better quality. This shifts preferences of domestic and foreigners' consumers towards home produced good. Such an increase in non-price competitiveness can be explained by labour productivity improvement in the open sector, because technology is usually imported from abroad via massive foreign direct investment (FDI), which is reflected in the productivity advances in the manufacturing sector. Based on econometric estimates, Oomes (2005) found cointegrating relationships between a number of real exchange rate measures and productivity differential for Slovakia. Beginning in 1994 until 1997, growth of equilibrium real effective exchange rate reflected productivity growth (Figure 2, Panel (g)). This process was halted in 1998 – 1999 only to be reinforced later. From early 2000s Slovakia with other Central European economies (Czech Republic, Hungary, and Poland) were rapidly integrated into the greater German supply chain.[19] FDIs (directed dominantly into automotive sector), as a percentage of GDP, soared in Slovakia from virtually zero in 2002 to more than 5% in 2006. This was a huge boost to productivity, which caused almost linear trend in the growth of equilibrium real exchange rate from 2000 to 2006. Growth in equilibrium level of real exchange rate peaked in tandem with productivity growth and it was well before the Global financial crisis, which can be related to the peak in FDIs in 2006. The Global financial crisis and the European debt crisis depressed productivity growth even more and since than we have not observed any trend appreciation of the equilibrium real effective exchange rate at all.

[19] Augustyniak et al. (2013).

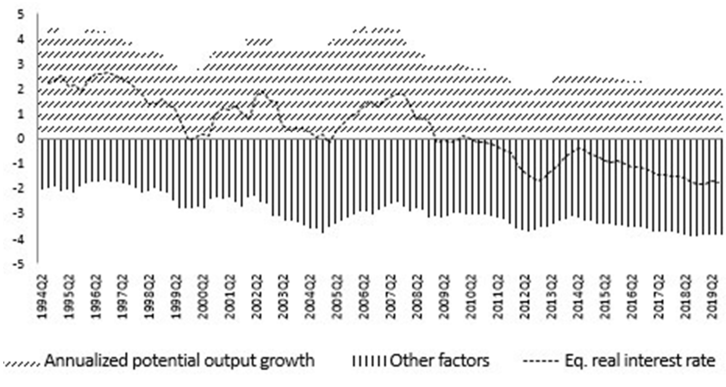
3.3.2. Equilibrium Real Interest Rate and the Real Interest Rate Gap Impulse

Aligned with many empirical studies (such as Rachel and Smith, 2017) documenting decline of the equilibrium real interest rate around the world, we have come to the similar conclusion in Slovakia. According to our estimates, r^* in Slovakia declined from the pre-crisis average value of 1.30% to the negative post-crisis average value of −0.92% (Figure 3).

As was correctly pointed in Benčík (2009b) in the sample until 2007: "*...that a process of convergence with the original European Union countries is going on in Slovakia and that within the process the so-called Balassa-Samuelson effect arises, causing pressure on the real exchange rate. Due to impossibility of appreciation of the nominal exchange rate following the introduction of the euro, these pressures will cause inflation in Slovakia to increase.*" This was correct observation. Average annual HICP inflation in the reflation periods (2011 – 2013 and 2017-present) was on average about 1% higher in Slovakia than in the Eurozone. With the same and low nominal interest rates within the Eurozone, this means lower and potentially more negative real rates in Slovakia than in the Euro area and, therefore, lower equilibrium rate. Again, this was correctly assumed in Benčík (2009b) and confirmed in later published estimates of r^* for EA,[20] which show on average higher values from 2009 than the ones estimated here for Slovakia.

Figure 3

Equilibrium Real Interest Rate and Its Components



Source: Author's own computations.

Based on the neoclassical growth model (see equation (8a) and (8b)), there are two main driving forces behind this fall globally, such as slowdown in growth, and shifts in preferences for savings and investment. Regarding the first

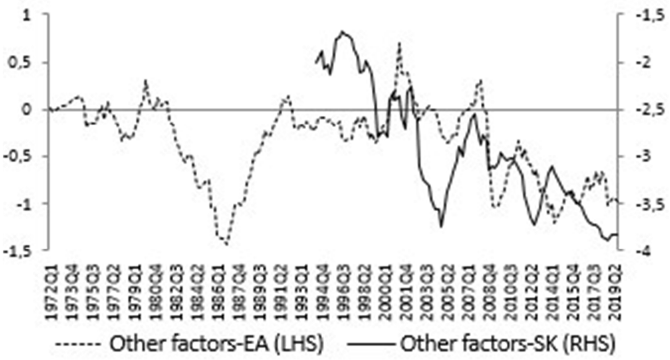
[20] See for example Holston, Laubach and Williams (2017) or Brand and Mazelis (2019).

factor, Rachel and Smith (2017) decompose growth component into three sub-components. The first one is related to growth of labour supply. Globally, world has experienced its peak rate of working age population growth in 70s – 80s and the trend is one of slowing population growth.

In Slovakia, the annual working age population growth from 1993 to 2002 was around 0.9%, then from 2002 to 2012 slipped to zero, and in 2018 was negative at –0.8%. The second one is catch-up growth. On average, its contribution to the global slowdown in growth is neutral, however, in Slovakia played a huge role in 2002 – 2006 period as we discussed in the previous section. Finally, the progress at the technological frontier is slowing as is documented in Gordon (2014), which means decrease in adaptation of new technologies also in Slovakia.

F i g u r e 4

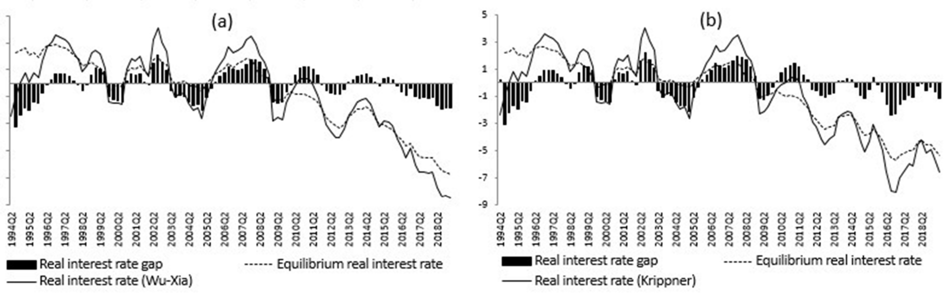
Other Factors in *r for Slovakia and Euro Area**



Source: Author’s own computations and Holston, Laubach and Williams (2017).

F i g u r e 5

Equilibrium Real Interest Rate under Non-standard Monetary Policy



Source: Author’s own computations.

As the shadow rates take into account non-standard monetary policy measures (see Figure 5, Panel (a) and (b)), resulting real interest rates are more negative after the Global financial crisis then the one which results from the standard policy rate (see Figure 2, Panel (c) and (d)). In terms of the real interest rate gap, the gap based on these measures has an average value of $\left(\tilde{r}_{2009}^{2019} = -0.6\%\right)$, and is more expansionary than the value which takes into account only standard monetary policy.

Conclusion

The equilibrium real interest rate is the main concept in the modern macroeconomic theory. Deviations of the real rate from its equilibrium influence the real economic activity which translates into price pressures. In the case of a small open economy, this framework is extended by the real exchange rate development. The aim of this paper was to develop a suitable framework for modelling the equilibrium real interest rates in transition economies. Contribution to the literature was in two ways: i) incorporating transition process in the model, and (ii) assessment of the adoption of the euro and its effect on the equilibrium real interest rate.

Regarding the transition process, the most dynamic periods were 1994 – 1997 and 2000 – 2006. Initial real undervaluation helped to boost convergence in the first period, whereas massive inflow of FDIs in the latter period lifted productivity and speed up the process as well. Adoption of the euro in 2009 led to the convergence of the nominal rates, and due to aggressive monetary policy easing, to the very low levels. This resulted in negative real and subsequently equilibrium real interest rates. These ideas could be applied to other transition economies, which entered the monetary union.

Empirical evidence suggests that the forces that have globally depressed real and potentially equilibrium real interest rates are likely to persist, and the equilibrium real rate may settle at low levels over the medium term. The policy implications of permanently low real rates are huge. Central banks are likely to be constrained by the zero-lower bound on nominal interest rates more often requiring the use of unconventional monetary policy instruments. On the other hand, the mainstream view used here and adopted among central banks is questioned at the BIS.[21] They argue that monetary policy may have played a more important role than commonly thought in long-run real economic outcomes, including real and equilibrium real interest rates. The link is the interaction between monetary policy and the financial cycle. Whether r^* is independent of monetary policy or is determined by the previous central bank decisions is an open issue and it is behind the scope of this paper.

[21] See Borio, Disyatat and Rungcharoenkitkul (2019).

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Appendix

Smoothed States with Non-standard Monetary Policy Measures

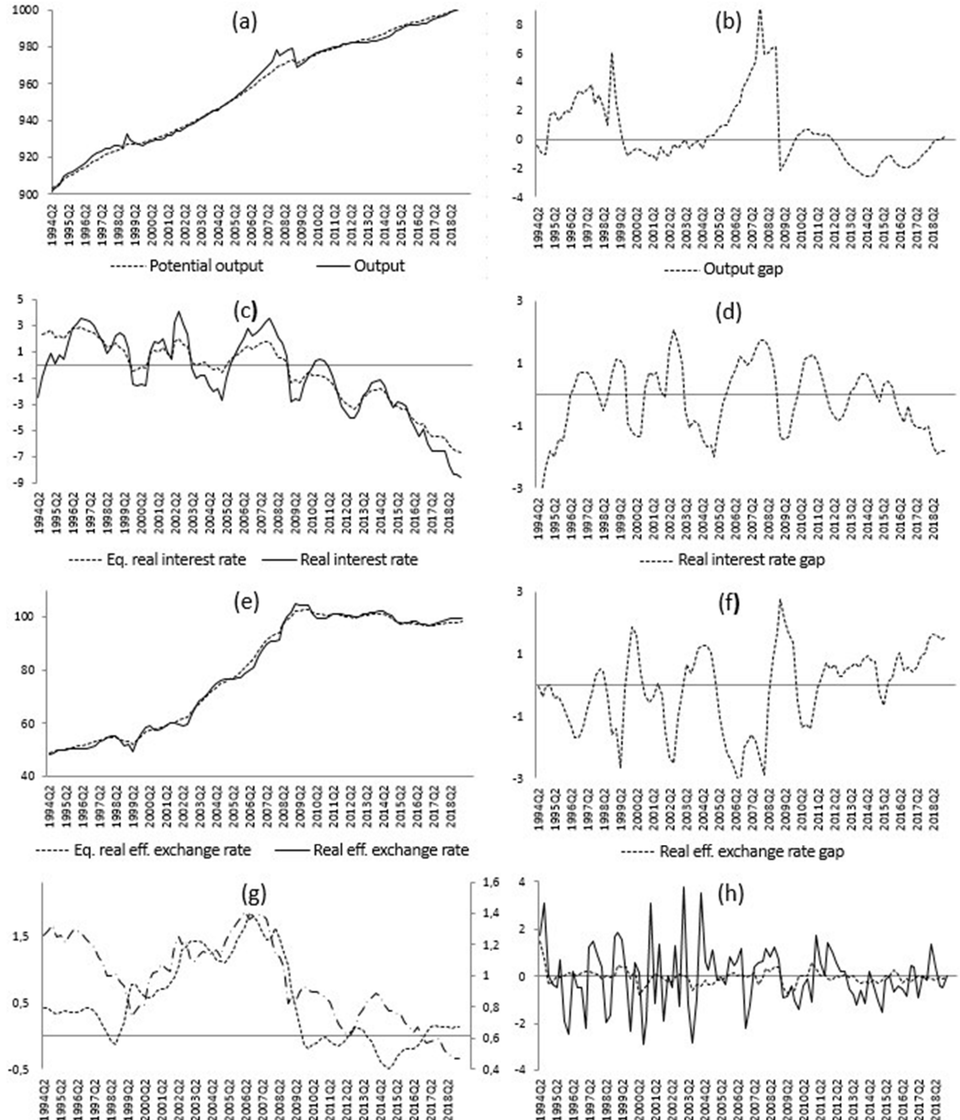
Table A.1
Prior and Posterior Parameter Distributions with Wu and Xia (2016)'S Shadow Policy Rate

	Prior distribution			Posterior distribution		
Parameter	*Type*	*Mean*	*s.d.*	*Mean*	*10pct.*	*90pct.*
α_y	Normal	0.750	0.200	0.788	0.597	0.982
α_r	Normal	–0.050	0.150	–0.140	–0.338	0.062
α_q	Normal	–0.250	0.150	–0.220	–0.423	–0.023
σ_{y^*}	Inv. gamma	0.750	0.025	0.753	0.710	0.793
σ_g	Inv. gamma	0.150	inf.	0.111	0.045	0.179
$\sigma_{\tilde{y}}$	Inv. gamma	2.000	inf.	1.370	1.132	1.610
β_y	Normal	0.250	0.150	0.074	–0.022	0.173
β_q	Normal	–0.250	0.150	–0.166	–0.296	–0.034
σ_π	Inv. gamma	3.000	inf.	1.274	1.274	1.420
c	Normal	4.000	1.000	3.607	1.973	5.158
γ	Normal	0.000	0.500	–0.441	–0.704	–0.173
ρ	Normal	0.750	0.200	0.829	0.699	0.999
σ_z	Inv. gamma	0.700	inf.	0.437	0.232	0.633
σ_κ	Inv. gamma	1.500	inf.	0.555	0.377	0.725
d_q	Normal	0.750	0.200	0.891	0.792	0.999
σ_{q^*}	Inv. gamma	1.200	inf.	0.677	0.375	0.959
σ_μ	Inv. gamma	0.400	inf.	0.310	0.144	0.486
$\sigma_{\tilde{q}}$	Inv. gamma	2.000	inf.	0.878	0.628	1.120

Source: Author's own computations, Berger and Kempa (2014).

Figure A.1

Smoothed States: Wu and Xia (2016)'s Shadow Policy Rate



Notes: Last data point 2019Q1.

Source: SO SR, NBS, ECB, EC, and author's own computations.

T a b l e A.2

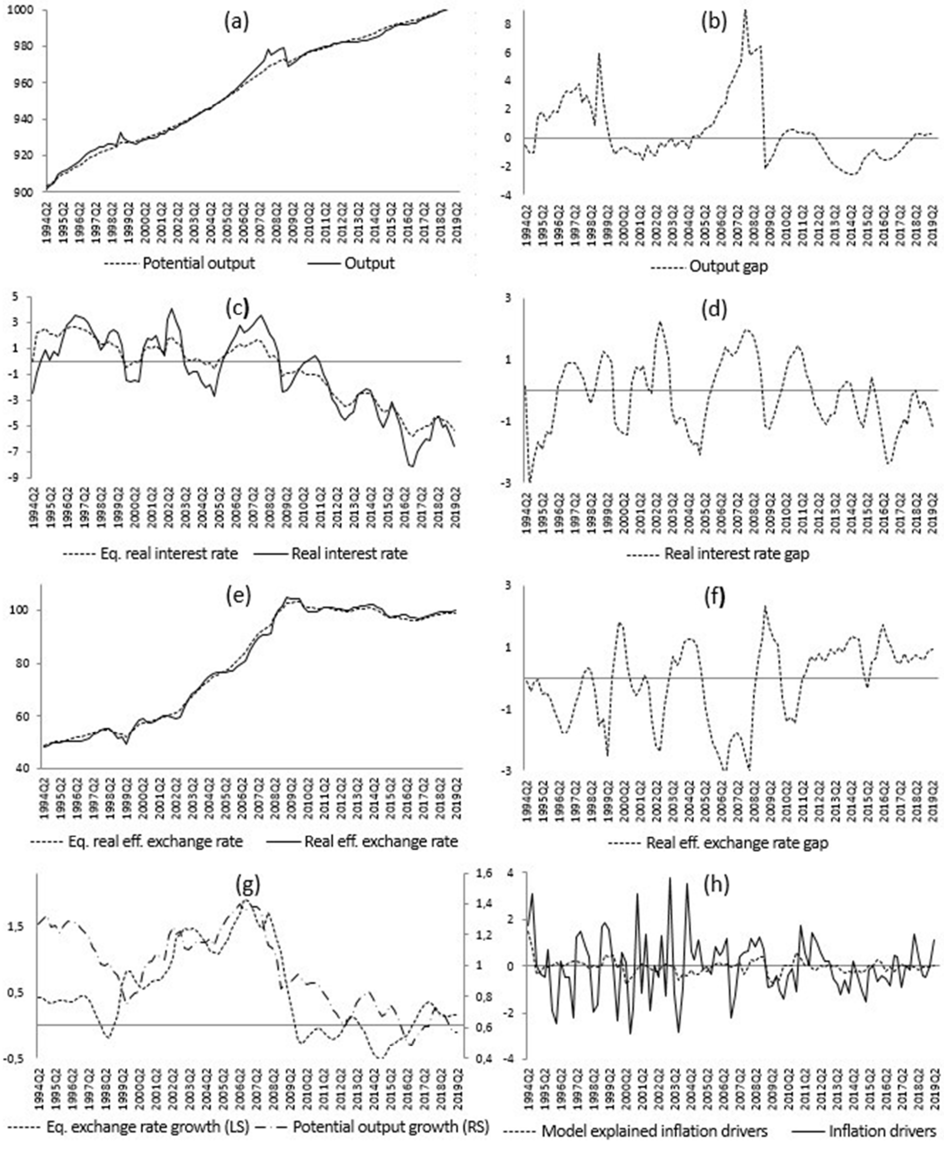
Prior and Posterior Parameter Distributions with Krippner (2013)'S Shadow Policy Rate

	Prior distribution			Posterior distribution		
Parameter	*Type*	*Mean*	*s.d.*	*Mean*	*10pct.*	*90pct.*
α_y	Normal	0.750	0.200	0.778	0.588	0.982
α_r	Normal	–0.050	0.150	–0.126	–0.321	0.066
α_q	Normal	–0.250	0.150	–0.235	–0.438	–0.032
σ_{y^*}	Inv. gamma	0.750	0.025	0.751	0.710	0.792
σ_g	Inv. gamma	0.150	inf.	0.112	0.044	0.180
$\sigma_{\tilde{y}}$	Inv. gamma	2.000	inf.	1.359	1.123	1.586
β_y	Normal	0.250	0.150	0.079	–0.016	0.180
β_q	Normal	–0.250	0.150	–0.165	–0.298	–0.035
σ_π	Inv. gamma	3.000	inf.	1.273	1.124	1.423
c	Normal	4.000	1.000	3.589	1.971	5.108
γ	Normal	0.000	0.500	–0.488	–0.774	–0.221
ρ	Normal	0.750	0.200	0.818	0.677	0.980
σ_z	Inv. gamma	0.700	inf.	0.417	0.224	0.612
σ_κ	Inv. gamma	1.500	inf.	0.580	0.400	0.756
d_q	Normal	0.750	0.200	0.888	0.788	0.999
σ_{q^*}	Inv. gamma	1.200	inf.	0.670	0.372	0.946
σ_μ	Inv. gamma	0.400	inf.	0.326	0.150	0.502
$\sigma_{\tilde{q}}$	Inv. gamma	2.000	inf.	0.865	0.618	1.094

Source: Author's own computations, Berger and Kempa (2014).

Figure A.2

Smoothed States: Krippner (2013)'s Shadow Policy Rate



Notes: Last data point 2019Q3.

Source: SO SR, NBS, ECB, EC, and author's own computations.